HORNSBY GIRLS HIGH SCHOOL

Mathematics

Year 12 Higher School Certificate
Trial Examination Term 3 2016

STUDENT NUMBER: ____________________

General Instructions

- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators and drawing templates may be used
- A reference sheet is provided separately
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for untidy and poorly arranged work
- Do not use correction fluid or tape
- Do not remove this paper from the examination

Total marks – 100

Section I Pages 3 – 6

10 marks

Attempt Questions 1 – 10

Answer on the Objective Response Answer Sheet provided

Section II Pages 7 – 15

90 marks

Attempt Questions 11 – 16

Start each question in a new writing booklet

Write your student number on every writing booklet

Question	***1-10***	***11***	***12***	***13***	***14***	***15***	***16***	***Total***
Total	*/10*	*/15*	*/15*	*/15*	*/15*	*/15*	*/15*	*/100*

This assessment task constitutes 45% of the Higher School Certificate Course School Assessment

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the Objective Response answer sheet for Questions 1 – 10

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1 What is the value of $\log_3 5$ correct to 4 significant figures?

(A) 1.465

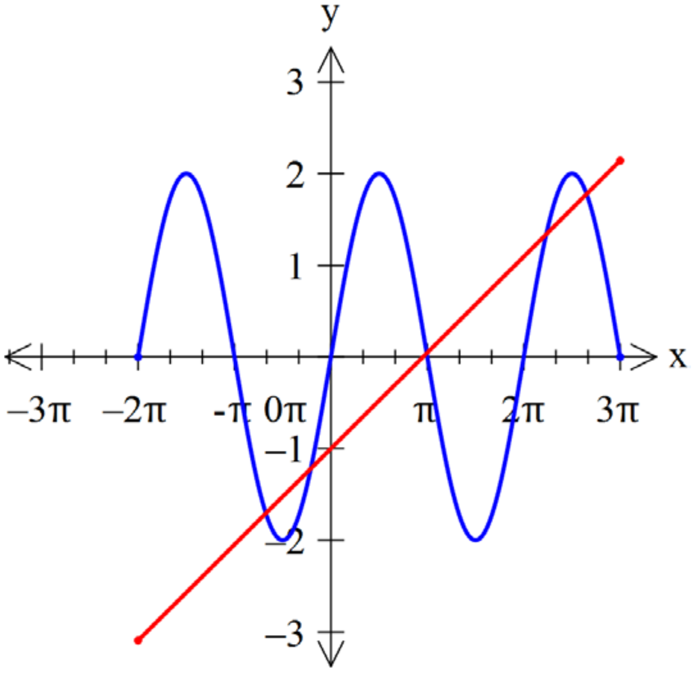
(B) 1.464

(C) 1.609

(D) 1.610

2 The graph below shows the curves $y = 2\sin x$ and $y = \dfrac{x}{3} - 1$ for $-2\pi \le x \le 3\pi$.

How many solutions does the equation $6\sin x + 3 = x$ have?



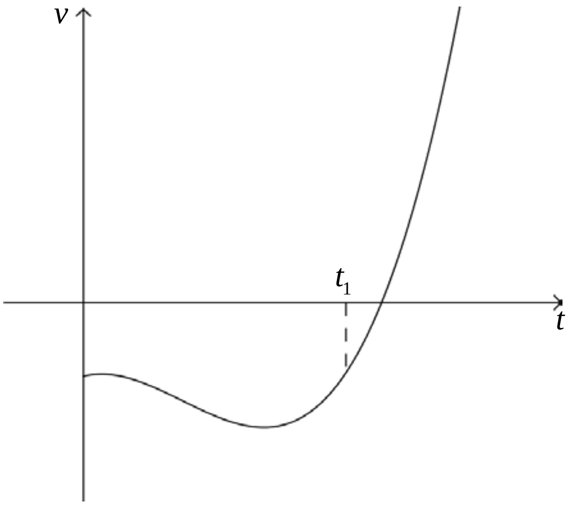
(A) 0

(B) 3

(C) 4

(D) 5

3 The graph below shows the velocity v of a particle moving along a straight line as a function of time t. The positive direction of motion is to the right.



Which statement describes the motion of the particle when $t = t_1$?

(A) The velocity is positive and acceleration is positive

(B) The velocity is negative and the acceleration is positive

(C) The velocity is positive and the particle is to the right of its initial position

(D) The velocity is negative and the particle is to the right of its initial position

4 The angle of inclination the line $3x + 2y - 7 = 0$ makes with the positive direction of the *x*-axis is closest to:

(A) 56°

(B) 124°

(C) 34°

(D) 146°

5 Which of the following is a term of the geometric series $-2x, 6x^3, -18x^5$ ... ?

(A) $4374x^{10}$

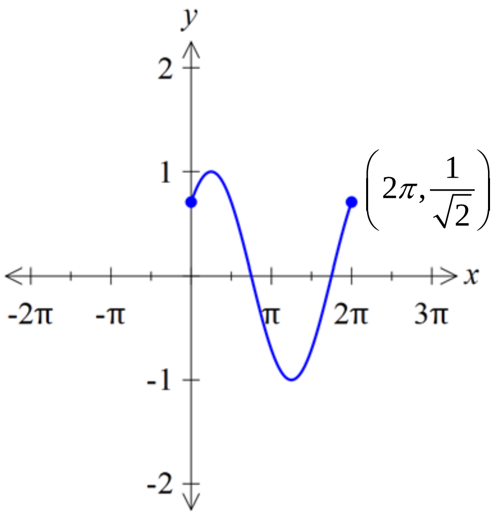
(B) $-4374x^{10}$

(C) $-4374x^{15}$

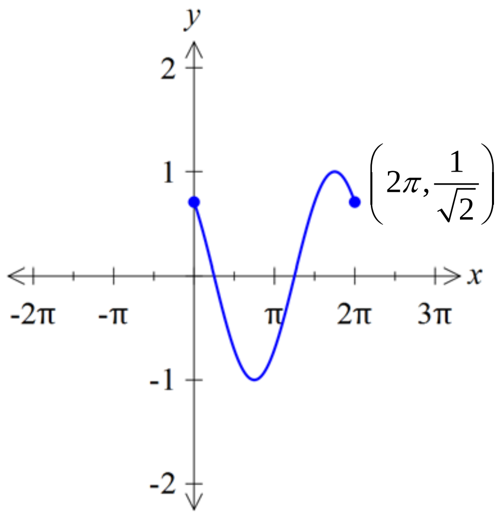
(D) $4374x^{15}$

6 The graph of $y=\sin\left(x+\frac{\pi}{4}\right)$ for $0\le x\le 2\pi$ is

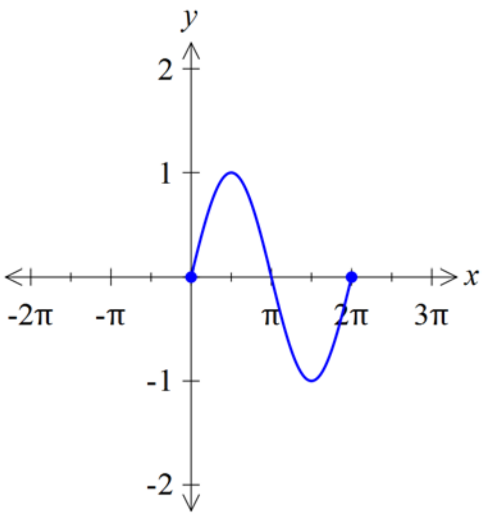
(A)



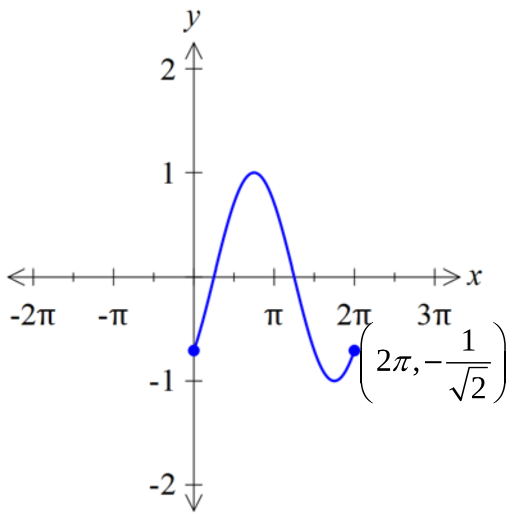
(B)



(C)



(D)



7 What is the primitive function of $\sec^2 2x$?

(A) $\tan x+C$

(B) $\tan 2x+C$

(C) $\frac{1}{2}\tan x+C$

(D) $\frac{1}{2}\tan 2x+C$

8 For what values of k does the quadratic equation $x^2 - 2kx + 4 = 0$ have no real solutions?

(A) $k > 2 \text{ or } k < -2$

(B) $-2 \le k \le 2$

(C) $|k| < 2$

(D) all values of k.

9 How many solutions does the equation $(2\sin x - 1)(\cos x + 1) = 0$ for $0 \le x \le 2\pi$ have?

(A) 2

(B) 3

(C) 4

(D) 5

10 The domain of the graph of $y = \log_2 |2x - 3|$ is:

(A) real x, $x > \frac{3}{2}$

(B) all real x, $x \ne \frac{3}{2}$

(C) $x > 0$

(D) all real x.

End of Section I

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question in a new writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new writing booklet

(a) Expand and simplify $(2x-y)(2x+y)-y(y-x)$. **2**

(b) Simplify $\dfrac{1}{1-\sqrt{2}}+\dfrac{4}{1+\sqrt{2}}$. **2**

(c) Differentiate $(3-x^2)^4$. **2**

(d) Evaluate $\displaystyle\int_0^{\frac{1}{2}} e^{2x}dx$. **2**

(e) If $\sin x=-\dfrac{1}{5}$, and $\cos x>0$, find the exact value of $\tan x$. **2**

(f) Evaluate $\displaystyle\sum_{n=1}^{5} 2n^2$. **2**

(g) If $\log_a 5=1.3$ and $\log_a 7=1.5$, find the value of:

(i) $\log_a 35$. **1**

(ii) $\log_a \dfrac{25}{7}$. **2**

Question 12 (15 marks) Start a new writing booklet

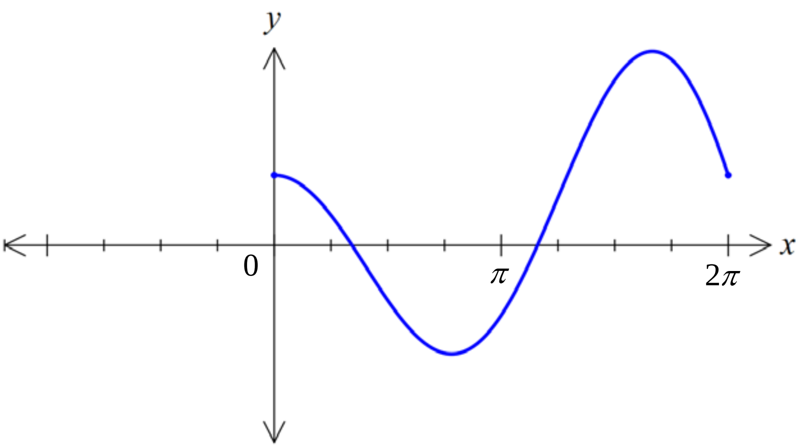
(a) Find the equation of the line passing through the point of intersection of $2x-3y-5=0$ and $x+2y=7$, and the point $(2,4)$. Express your answer in general form. **3**

(b) Consider $f(x)=x\sin x$.

(i) Show that $f''(x)=2\cos x-x\sin x$. **2**

(ii) Use the trapezoidal rule with 3 function values to approximate $\int_0^{\frac{\pi}{4}} x\sin x\,dx$, correct to 3 decimal places. **2**

(iii) The graph of $y=2\cos x-x\sin x$ is shown below. **1**



Is the approximation in part (ii) an overestimate or an underestimate?
Give reasons for your answer.

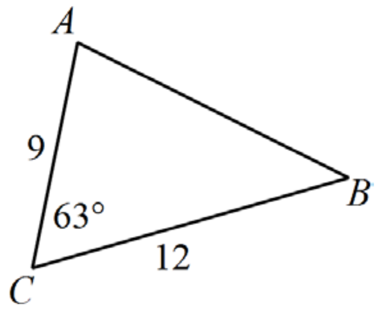
(c) The roots of the quadratic equation $2x^2-3x+1=0$ are α and β.
Find the value of

(i) $\alpha+\beta$. **1**

(ii) $\alpha^2+\beta^2$. **2**

Question 12 continues on page 9

(d) Find the length of AB in the triangle below, correct to one decimal place. **2**



NOT TO SCALE

(e) Find the limiting sum of the geometric series $2-\frac{2}{3}+\frac{2}{9}-\frac{2}{27}+...$ **2**

End of Question 12

Question 13 (15 marks) Start a new writing booklet

(a) (i) Sketch the parabola P whose focus is the point $(2,3)$ and whose directrix is the line $y=-1$. Indicate on your diagram the vertex and its coordinates. **2**

(ii) Find the equation of P. **1**

(b) A particle moves along the x-axis in such a way that its position at time t is given by $x=2t^3-15t^2+24t+3$, where x is in metres and t is in seconds.

(i) Determine the velocity and acceleration of the particle at time t. **2**

(ii) At what values of t is the particle at rest? **1**

(iii) What is the velocity when the acceleration is first zero? **1**

(iv) Calculate the distance travelled by the particle in the first 3 seconds. **2**

(c) George takes out a loan from the bank for \$80 000 for house renovations. It is to be repaid by monthly repayments of \$500, with the first repayment taking place at the end of the first month.

Interest is charged monthly on the balance owing and then the repayment is made.
The interest rate is 6% per annum, compounded monthly.

Let \$ A_n be the amount owing after n months.

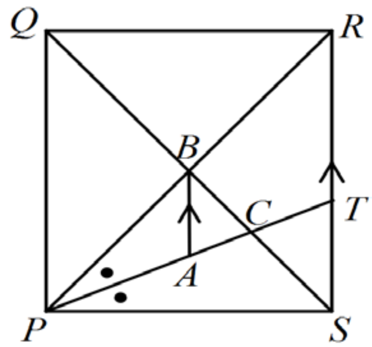
(i) Show that $A_3=80000\times1.005^3-500\left(1+1.005+1.005^2\right)$. **1**

(ii) Find how many months it takes for George to repay the loan, giving your answer to the nearest whole month. **3**

(iii) George wants to change his repayment at the beginning of the loan so that he can repay the loan in 5 years. Find the monthly repayment he needs to make for this to occur. **2**

Question 14 (15 marks) Start a new writing booklet

(a) In the figure, $PQRS$ is a square. QS and PR are diagonals and PT bisects $\angle SPR$ and $AB \parallel SR$.



NOT TO SCALE

Copy or trace the diagram into your writing booklet.

(i) Prove that $AB = BC$. **2**

(ii) Prove that $\Delta PBA \parallel\!| \Delta PRT$. **2**

(iii) Prove that $RT = 2BC$. **2**

(b) The population of cockroaches in Hornsby grows in such a way that the rate of change of the population P at time t in days is proportional to P, that is $\frac{dP}{dt} = kP$.

Initially there were 8 million cockroaches in Hornsby and 7 days later the population had grown to 8.5 million.

(i) Show that $P = Ae^{kt}$ is a solution to the differential equation. **1**

(ii) Find the size of the population 28 days after the population was 8.5 million. **3**

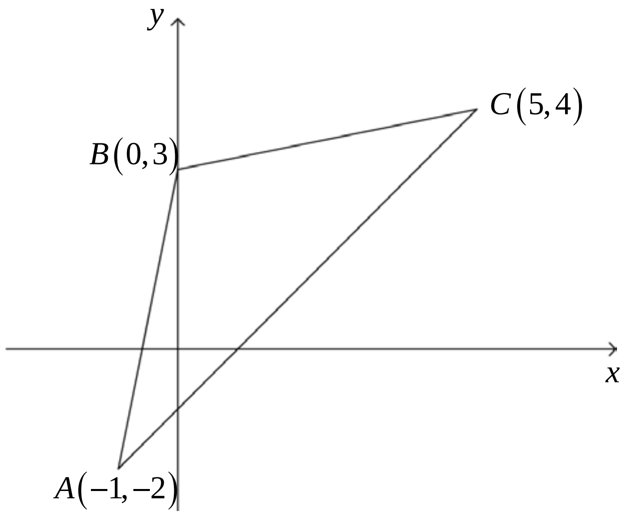
(iii) By what percentage does the population of the cockroaches increase by each day? **1**

Answer correct to the nearest 0.1%.

Question 14 continues on page 12

Question 14 (continued)

(c) The diagram below shows ΔABC with vertices $A(-1,-2)$, $B(0,3)$ and $C(5,4)$.



NOT TO SCALE

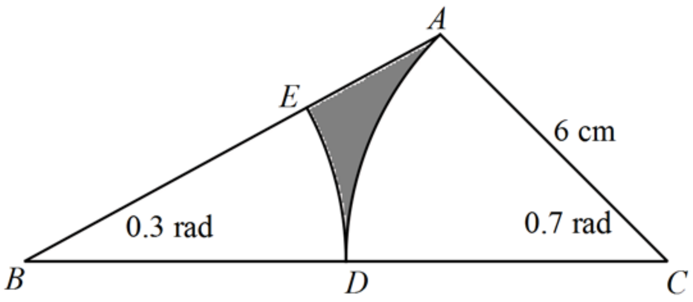
(i) Find the equation of AC. **2**

(ii) Find the exact area of ΔABC. **2**

End of Question 14

Question 15 (15 marks) Start a new writing booklet

(a) The diagram below shows the triangle ABC, with $AC = 6$ cm, $\angle ABC = 0.3$ radians and $\angle ACB = 0.7$ radians. The arc AD, where D lies on BC, is an arc of a circle with centre C and radius 6 cm. The arc DE, where E lies on AB, is an arc of a circle with centre B.
The shaded region is bounded by straight lines EA and the arcs AD and DE.



NOT TO SCALE

(i) Show that $BC = 17.1$ cm, correct to 1 decimal place. **2**

(ii) Using $BC = 17.1$ cm, find the length of arc ED, correct to 1 decimal place. **2**

(iii) Using $BC = 17.1$ cm and your result from part (ii), find the area of shaded region, giving your answer correct to 1 decimal place. **2**

(b) The curve C with equation $y = f(x)$ passes through the point $(2,4)$ and has first derivative $f'(x) = 3(x-1)(x+1)$.

(i) Use integration to find $f(x)$. **2**

(ii) By expanding $(x-1)^2(x+2)$, show that this expression is equivalent to your expression for $f(x)$ from part (i). **1**

(iii) Find the coordinates of the stationary points of $y = f(x)$ and determine their nature. **2**

(iv) Find the coordinates of any points of inflexion of $y = f(x)$. **2**

(v) Sketch the graph of $y = f(x)$ showing the stationary points, points of inflexion and intercepts. **2**

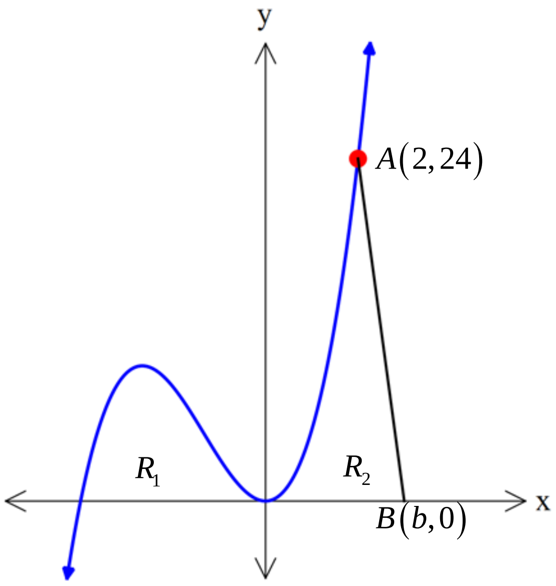
Question 16 (15 marks) Start a new writing booklet

(a) A language class has students who study at least one of the languages French and German. **2**
Of the class of 30 students, 16 study German and 26 study French.
What is the probability that a student selected at random studies both languages?

(b) The sketch below shows part of the curve with equation $y = x^2(x+4)$.

The finite region R_1 is bounded by the curve and the negative x-axis.

The finite region R_2 is bounded by the curve, the positive x-axis and AB, where $A=(2,24)$ and $B(b,0)$ where $b>2$.



NOT TO SCALE

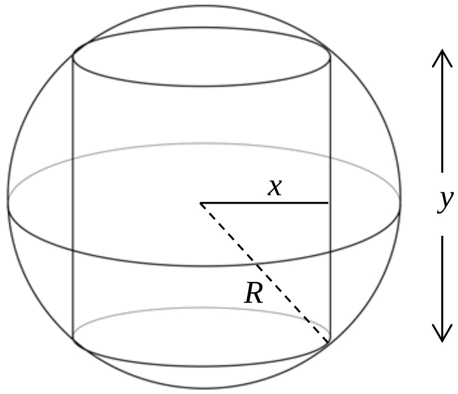
(i) Show that the area of R_1 is $\frac{64}{3}$ square units. **2**

(ii) If the area of the regions R_1 and R_2 are equal, find the exact value of b. **3**

Question 16 continues on page 15

Question 16 (continued)

(c) A right cylinder of radius x and height y is inscribed in a sphere of radius R, where R is a constant.



The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

The volume of a right cylinder with radius r and height h is $\pi r^2 h$.

(i) Show that the volume of the cylinder V, can be written as $V = \pi R^2 y - \frac{\pi y^3}{4}$. **2**

(ii) Prove that the maximum volume of the cylinder V occurs when $y = \frac{2R}{\sqrt{3}}$. **3**

(iii) Find the maximum volume of the cylinder V in terms of R in simplest form. **2**

(iv) When the cylinder has a maximum volume, show that the ratio of the volume of the cylinder to the volume of the sphere is $1:\sqrt{3}$. **1**

End of Paper

Year 12 Mathematics 2 unit Trial Examination Solutions 2016

Multiple Choice

Q1.

$\log_3 5 = \frac{\ln 5}{\ln 3}$

$= 1.46497...$

$= 1.465\ (4sf)$

(A)

Q2.

$2\sin x = \frac{x}{3} - 1$

$6\sin x = x - 3$

$x = 3 + 6\sin x$

Five points of intersection

(D)

Q3.

The velocity is negative. The particle has been travelling in the negative direction.

Therefore the displacement is negative. And the acceleration is positive as velocity is increasing.

(B)

Q4.

$3x + 2y - 7 = 0$

$2y = -3x + 7$

$y = -\frac{3}{2}x + \frac{7}{2}$

$\theta = \tan^{-1}\left(\frac{-3}{2}\right)$

$= 124°$

(B)

Q5.

Powers are going up by two, so are odd. Term with power of 15th will be positive

(D)

Q6.

(A)

Q7.

$\int \sec^2 2x dx = \frac{1}{2}\tan 2x + C$

(D)

Q8.

$\Delta < 0$

$4^2 - 16 < 0$

$k^2 - 4 < 0$

$-2 < k < 2$

(C)

Q9.

$\sin x = \frac{1}{2}$ $\qquad \cos x = -1$

$x = \frac{\pi}{5}, \frac{5\pi}{6}$ $\qquad x = -\pi$

Therefore 3 solutions

(B)

Q10.

$|2x - 3| > 0\ for\ all\ x, x \neq \frac{3}{2}$

(B)

Question 11

(a)

$$(2x-y)(2x+y)-y(y-x)$$
$$=(2x)^2-y^2-y^2+xy$$
$$=4x^2-2y^2+xy$$

(b)

$$\frac{1}{1-\sqrt{2}}+\frac{4}{1+\sqrt{2}}=\frac{1+\sqrt{2}}{1-2}+\frac{4(1-\sqrt{2})}{1-2}$$
$$=\frac{1+\sqrt{2}+4-4\sqrt{2}}{-1}$$
$$=\frac{5-3\sqrt{2}}{-1}$$
$$=3\sqrt{2}-5$$

(c)

$$f(x)=(3-x^2)^4$$
$$f'(x)=4(3-x^2)^3\times(-2x)$$
$$=-8x(3-x^2)^3$$

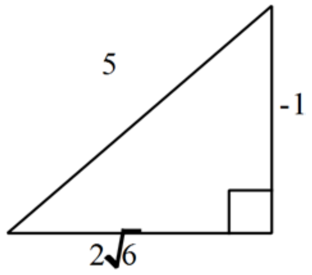
(d)

$$\int_0^{\frac{1}{2}} e^{2x}dx=\left[\frac{e^{2x}}{2}\right]_0^{\frac{1}{2}}$$
$$=\left[\frac{e^{2\left(\frac{1}{2}\right)}}{2}\right]-\left[\frac{e^{2(0)}}{2}\right]$$
$$=\frac{e}{2}-\frac{1}{2}$$
$$=\frac{1}{2}(e-1)$$

(e)

$\sin x=-\frac{1}{5}$, $\cos x>0$

$\therefore$ x is in the 4th quadrant



$$\tan x=\frac{-1}{2\sqrt{6}}$$
$$=\frac{-\sqrt{6}}{12}$$

(f)

$$\sum_{n=1}^{5}2n^2=2(1^2+2^2+3^2+4^2+5^2)$$
$$=2(1+4+9+16+25)$$
$$=110$$

(g)

(i)

$$\log_a 35=\log_a(5\times7)$$
$$=\log_a 5+\log_a 7$$
$$=1.3+1.5$$
$$=2.8$$

(ii)

$\log_a 5=1.3$ $\quad$ $\log_a 7=1.5$

$$\log_a\frac{25}{7}=\log_a 5^2-\log_a 7$$
$$=2\log_a 5-\log_a 7$$
$$=2(1.3)-1.5$$
$$=2.6-1.5$$
$$=1.1$$

Question 12

(a)

$$2x-3y-5=0$$
$$x+2y=7$$
$$x+2y-7=0$$

$$(2x-3y-5)+k(x+2y-7)=0$$
$$Sub\ x=2,\ y=4$$
$$(2(2)-3(4)-5)+k(2+2(4)-7)=0$$
$$4-12-5+k(2+8-7)=0$$
$$-13+3k=0$$
$$3k=13$$
$$k=\frac{13}{3}$$

The equation of the line is:

$$(2x-3y-5)+\frac{13}{3}(x+2y-7)=0$$
$$6x-9y-15+13x+26y-91=0$$
$$19x+17y-106=0$$

(b)

(i)

$$f(x)=x\sin x$$
$$f'(x)=1\times\sin x+x\times\cos x$$
$$=\sin x+x\cos x$$
$$f''(x)=\cos x+1\times\cos x+x\times-\sin x$$
$$=2\cos x-x\sin x$$

(ii)

Trapezoidal rule with three function values

$$\int_0^{\frac{\pi}{4}} x\sin x\,dx\approx\frac{\frac{\pi}{8}}{2}\left(f(0)+2f\left(\frac{\pi}{8}\right)+f\left(\frac{\pi}{4}\right)\right)$$
$$=\frac{\pi}{16}\left(0+\frac{\pi}{4}\sin\frac{\pi}{8}+\frac{\pi}{4}\sin\frac{\pi}{4}\right)$$
$$=0.168059...$$
$$\approx 0.168\,(3sf)$$

(iii)

Since graphically shown that $2\cos x-\sin x>0$ for $0\le x\le\frac{\pi}{4}$, the concavity of $y=x\sin x$ is concave up in this interval. Therefore the use of the trapezoidal rule is an overestimation

(c)

$$2x^2-3x+1=0$$

(i)

$$\alpha+\beta=\frac{-b}{a}$$
$$=-\frac{(-3)}{2}$$
$$=\frac{3}{2}$$

(ii)

$$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$$
$$=\left(\frac{3}{2}\right)^2-2\left(\frac{c}{a}\right)$$
$$=\frac{9}{4}-2\left(\frac{1}{2}\right)$$
$$=\frac{9}{4}-1$$
$$=\frac{5}{4}$$

(d)

By cosine rule:

$$AB^2=CA^2+AB^2-2CA.CB.\cos 63°$$
$$=9^2+12^2-2(9)(12)\cos 63°$$
$$=126.9380521...$$
$$AB=11.2666..$$
$$=11.3\text{ units}\,(1dp)$$

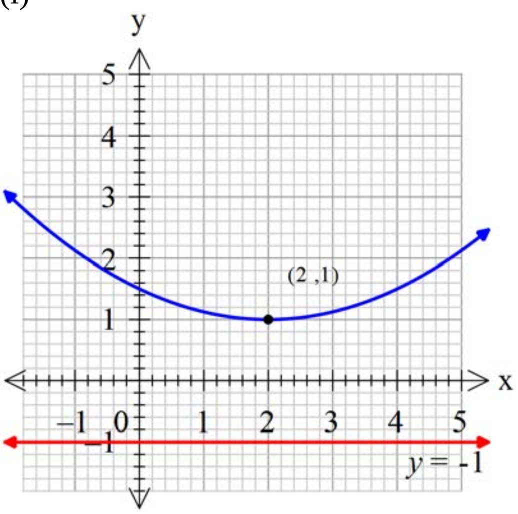
(e)

$$2-\frac{2}{3}+\frac{2}{9}-\frac{2}{27}+...$$
$$a=2$$
$$r=\frac{-1}{3}$$
$$S_\infty=\frac{a}{1-r}$$
$$=\frac{2}{1-\left(\frac{-1}{3}\right)}$$
$$=\frac{2}{\frac{4}{3}}$$
$$=\frac{3}{2}$$

Question 13

(a)

(i)



(ii)

Focal length is 2

Vertex is $(2,1)$

Equation of P:

$(x-2)^2 = 4\times 2(y-1)$

$(x-2)^2 = 8(y-1)$

(b)

$x = 2t^3 - 15t^2 + 24t + 3$

(i)

$\frac{dx}{dt} = 6t^2 - 30t + 24$

$\frac{d^2x}{dt^2} = 12t - 30$

(ii)

Let $\frac{dx}{dt} = 0$

$6t^2 - 30t + 24 = 0$

$t^2 - 5t + 4 = 0$

$(t-4)(t-1) = 0$

$t = 4 \text{ or } t = 1$

$\therefore$ The particle is at rest at 1 and 4 seconds.

(iii)

$Let \ \frac{d^2x}{dt^2} = 0$

$12t - 30 = 0$

$12t = 30$

$t = \frac{5}{2}$

Sub into $\frac{dx}{dt}$

$\frac{dx}{dt} = 6\left(\frac{5}{2}\right)^2 - 30\left(\frac{5}{2}\right) + 24$

$= -13.5 \ ms^{-1}$

(iv)

When $t = 0$

$x = 3$

When $t = 1$

$x = 2 - 15 + 24 + 3$

$= 14$

When $t = 3$

$x = 2\times(3)^3 - 15\times(3)^2 + 24\times(3) + 3$

$= -6$

Total distance travelled is $(14-3)+14+6$

$= 31 \ metres$

(c)

(i)

Let $\$A_n$ be the amount owing after *n* months

$A_1 = 80000(1+0.005) - 500$

$= 80000\times 1.005 - 500$

$A_2 = A_1 \times 1.005 - 500$

$= 80000\times 1.005^2 - 500\times 1.005 - 500$

$= 80000\times 1.005^2 - 500(1+1.005)$

$A_3 = A_2 \times 1.005 - 500$

$= 80000\times 1.005^3 - 500\times 1.005\times(1+1.005) - 500$

$= 80000\times 1.005^3 - 500(1+1.005+1.005^2)$

(ii)

$A_n = 80000\times 1.005^n - 500(1+1.005+1.005^2+\ldots+1.005^{n-1})$

$= 80000\times 1.005^n - 500\left(\frac{1.005^n - 1}{1.005 - 1}\right)$

$= 80000\times 1.005^n - 100000(1.005^n - 1)$

$Let \ A_n = 0$

$100000 = 20000\times 1.005^n$

$\ln 5 = n \ln 1.005$

$n = \frac{\ln 5}{\ln 1.005}$

$= 322.69 \ (2dp)$

It takes George 323 months to repay the loan.

(iii)

$$A_n = 80000(1.005)^{60} - \frac{M\left(1.005^{60}-1\right)}{1.005-1} = 0$$

$$400(1.005)^{60} - M\left(1.005^{60}-1\right) = 0$$

$$-M\left(1.005^{60}-1\right) = -400(1.005)^{60}$$

$$M = \frac{400(1.005)^{60}}{1.005^{60}-1}$$

$$= \$1546.62$$

To repay the loan in 5 years the monthly repayments need to be \$1546.62, correct to the nearest cent.

Question 14

(a)

(i)

$Let\ \angle BPA = \alpha$

$In\ \Delta PBC, \angle PBC = 90°\ \ (diagonals\ of\ a\ square\ are\ perpendicular)$

$\angle BCA = 180° - 90° - \alpha\ \ (\angle\ sum\ of\ triangle)$

$= 90° - \alpha$

$\angle TSP = 90°\ (property\ of\ a\ square)$

$\therefore \angle RTP = 90° + \alpha\ \ (exterior\ \angle\ of\ triangle\ PTS)$

$\angle BAC + \angle RTP = 180°\ \ (co-interior\ \angle's,\ BA \parallel RT)$

$\angle BAC = 180° - (90° + \alpha)$

$= 90° - \alpha$

$= \angle BCA$

$\therefore AB = BC\ (equal\ sides\ opposite\ angles\ in\ a\ triangle)$

(ii)

In ΔPBA and ΔPRT

1. $\angle RPT$ is common.
2. $\angle PAB = \angle PTR\ \ (corresponding\ angles,\ AB \parallel RT)$

$\therefore \Delta PBA\ |||\ \Delta PRT\ (equiangular)$

(iii)

B is the midpoint of PR

$2PB = PR$

$$\therefore \frac{RT}{AB} = \frac{PR}{PB}\ \left(\begin{matrix} matching\ sides\ are\ in\ the\ same\ ratio, \\ \Delta PBA\ |||\ \Delta PRT \end{matrix}\right)$$

$$\frac{RT}{AB} = \frac{2PB}{PB}$$

$$\frac{RT}{AB} = 2$$

$$RT = 2AB$$

$$= 2BC$$

(b)

$$\frac{dP}{dt} = kP$$

(i)

$$P = Ae^{kt}$$

$$\frac{dP}{dt} = k \times Ae^{kt}$$

$$= k\left(Ae^{kt}\right)$$

$$= kP$$

(ii)

At $t=0$,

$P=8\times10^{6}$

$8\times10^{6}=Ae^{k\times0}$

$A=8\times10^{6}$

$\therefore P=8\times10^{6}e^{kt}$

At $t=7$, $P=8.5\times10^{6}$

$8.5\times10^{6}=8\times10^{6}e^{7k}$

$\frac{8.5}{8}=e^{7k}$

$7k=\ln\frac{8.5}{8}$

$k=\frac{1}{7}\ln\frac{8.5}{8}$

At $t=35$,

$P=8\times10^{6}\times e\frac{35k}{}$

$=8\times10^{6}\times e^{35\times\frac{1}{7}\ln\frac{8.5}{8}}$

$=1.08\times10^{7}$ $(3\,s\ f)$

(iii)

$k=0.008660...$ growth rate per day.

$\therefore 0.9\%$

(c)

$m=\frac{4+2}{5+1}$

$=1$

Equation of AC:

$y-4=1(x-5)$

$y=x-5+4$

$x-y-1=0$

(ii)

$AC^{2}=(5+1)^{2}+(4+2)^{2}$

$=36+36$

$=72$

$AC=\sqrt{72}$

$=6\sqrt{2}$

Distance of $B(0,3)$ from AC

$d=\frac{|0-3-1|}{\sqrt{1^{2}+(-1)^{2}}}$

$=\frac{4}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$

$=2\sqrt{2}$ units

$A=\frac{1}{2}\times6\sqrt{2}\times2\sqrt{2}$

$=12\,units^{2}$

Question 15

(a)

(i)

$$\angle BAC = \pi - (0.7 + 0.3)$$
$$= (\pi - 1)\, rad$$

By Sine Rule:

$$\frac{BC}{\sin A} = \frac{AC}{\sin B}$$
$$\frac{BC}{\sin(\pi - 1)} = \frac{6}{\sin 0.3}$$
$$BC = \frac{6\sin(\pi - 1)}{\sin 0.3}$$
$$= 17.0845...$$
$$= 17.1\ (to\ 1\ dp)$$

(ii)

$$BD = 17.1 - 6$$
$$= 11.1$$
$$arc\ ED = BD \times \theta$$
$$= 11.1 \times 0.3$$
$$= 3.3\ \text{cm}\ (1dp)$$

(iii)

$$A_{\Delta ABC} = \frac{1}{2}.BC.AC \sin 0.7$$
$$= \frac{1}{2}(17.1)(6)\sin 0.7$$
$$A_{\text{sector } BED} = \frac{1}{2} BD^2 \times 0.3$$
$$= \frac{1}{2} \times (11.1)^2 \times 0.3$$
$$A_{\text{sector CAD}} = \frac{1}{2} AC^2 \times 0.7$$
$$= \frac{1}{2}(6)^2 \times 0.7$$

$$Shaded\ Area = A_{\Delta ABC} - A_{\text{sector } BED} - A_{\text{sector CAD}}$$
$$= \frac{1}{2}(17.1)(6)\sin 0.7 - \frac{1}{2} \times (11.1)^2 \times 0.3$$
$$- \frac{1}{2}(6)^2 \times 0.7$$
$$= 1.9668...$$
$$= 2.0\ cm^2 \quad (1dp)$$

(b)

$$f'(x) = 3(x-1)(x+1)$$
$$f(x) = \int 3(x-1)(x+1)dx$$
$$= 3\int (x^2 - 1)dx$$
$$= 3\left(\frac{x^3}{3} - x\right) + C$$
$$= x^3 - 3x + C$$

At $(2,4)$,

$$4 = (2)^3 - 3(2) + C$$
$$4 = 8 - 6 + C$$
$$C = 2$$
$$\therefore f(x) = x^3 - 3x + 2$$

(ii)

$$(x-1)^2(x+2) = (x^2 - 2x + 1)(x+2)$$
$$= x^3 - 2x^2 + x + 2x^2 - 4x + 2$$
$$= x^3 + x - 4x + 2$$
$$= x^3 - 3x + 2$$
$$= f(x)$$

(iii)

$$Given\ f'(x) = 3(x-1)(x+1)$$
$$Let\ f'(x) = 0$$
$$\therefore x = -1\ or\ x = 1$$
$$f(-1) = [(-1) - 1]^2 [(-1) + 2]$$
$$= (-2)^2 \times 1$$
$$= 4$$
$$\therefore (-1, 4)$$
$$f(1) = [(1) - 1]^2 [(1) + 2]$$
$$= 0$$
$$\therefore (1, 0)$$

$$f'(x) = 3(x-1)(x+1)$$
$$= 3x^2 - 3$$
$$f''(x) = 6x$$
$$f''(-1) = 6(-1)$$
$$= -6$$
$\therefore (-1,4)$ is a maximum turning point

$$f''(1) = 6(1)$$
$$= 6$$
$\therefore (1,0)$ is a minimum turning point

(iv)

For point of inflexion, let $f''(x) = 0$

$6x = 0$

$x = 0$

$$f(0) = [(0)-1]^2[(0)+2]$$
$$= (1)(2)$$
$$= 2$$
$$\therefore (0,2)$$

Testing concavity change:

x	-0.1	0	0.1
$f''(x)$	-0.6	0	0.6

Therefore there is a change in concavity.

$\therefore (0,2)$ is a point of inflexion.

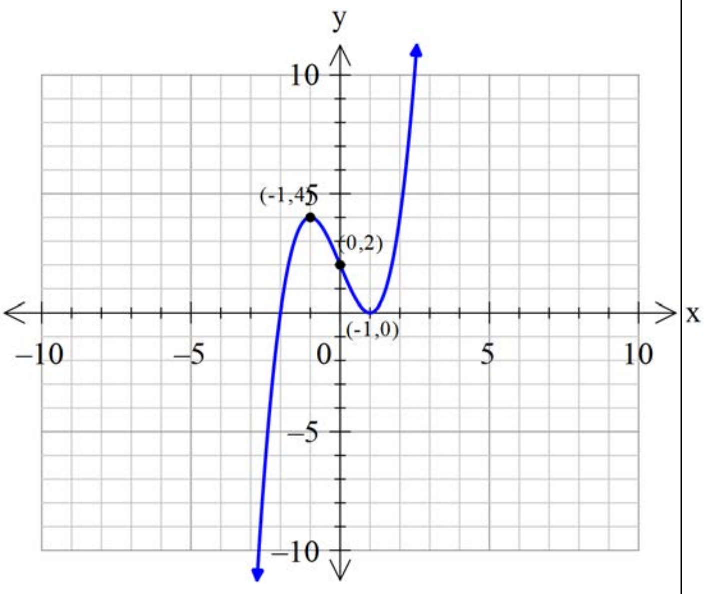
(v)

When $f(x) = 0$,

$(x-1)^2(x+2) = 0$

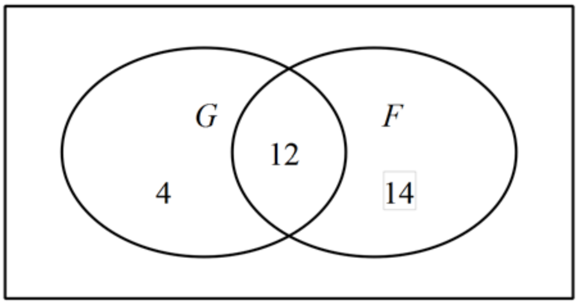
$x = 1, x = -2$

$f(0) = 2$



Question 16

(a)



$$P(\text{study both languages}) = \frac{12}{30} = \frac{2}{5}$$

(b)

(i)

$$\text{Area of } R_1 = \int_{-4}^{0} x^2(x+4)\,dx$$

$$= \int_{-4}^{0} (x^3 + 4x^2)\,dx$$

$$= \left[\frac{x^4}{4} + \frac{4x^3}{3}\right]_{-4}^{0}$$

$$= (0) - \left(64 - \frac{256}{3}\right)$$

$$= \frac{64}{3} \text{ units}^3$$

(ii)

$$R_2 = \frac{64}{3}$$

$$\frac{64}{3} = \int_0^2 (x^3 + 4x^2)\,dx + \frac{1}{2} \times 24 \times (b-2)$$

$$= \left[\frac{x^4}{4} + \frac{4x^3}{3}\right]_0^2 + 12(b-2)$$

$$= \frac{2^4}{4} + \frac{4(2)^3}{3} + 12b - 24$$

$$= 12b - \frac{28}{3}$$

$$12b = \frac{64}{3} + \frac{28}{3}$$

$$12b = \frac{92}{3}$$

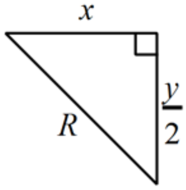
$$b = \frac{23}{9}$$

(c)

(i)

$$V = \pi r^2 h$$

$$= \pi x^2 y$$



$$R^2 = x^2 + \left(\frac{y}{2}\right)^2$$

$$x^2 = R^2 - \frac{y^2}{4}$$

$$\therefore V = \pi\left(R^2 - \frac{y^2}{4}\right)y$$

$$= \pi R^2 y - \frac{\pi y^3}{4}$$

(ii)

$$\frac{dV}{dy} = \pi R^2 - \frac{3\pi y^2}{4}$$

$$\frac{d^2V}{dy^2} = \frac{-6\pi y}{4}$$

$$= \frac{-3\pi y}{2}$$

Let $\frac{dV}{dy} = 0$

$$\pi R^2 - \frac{3\pi y^2}{4} = 0$$

$$\frac{3\pi y^2}{4} = \pi R^2$$

$$y^2 = \frac{4R^2}{3}$$

$$y = \frac{2R}{\sqrt{3}} \; (y > 0)$$

Since $\frac{d^2V}{dy^2} < 0$ since $y > 0$,

$y = \frac{2R}{\sqrt{3}}$ gives the maximum volume of the cylinder.

(iii)

Substitute $y = \frac{2R}{\sqrt{3}}$

$$V = \pi R^2 \times \frac{2R}{\sqrt{3}} - \frac{\pi}{4}\left(\frac{2R}{\sqrt{3}}\right)^3$$

$$= \frac{2\pi R^3}{\sqrt{3}} - \frac{\pi}{4} \times \frac{8R^3}{3\sqrt{3}}$$

$$= \frac{2\pi R^3}{\sqrt{3}} - \frac{2\pi R^3}{3\sqrt{3}}$$

$$= \frac{6\pi R^3 - 2\pi R^3}{3\sqrt{3}}$$

$$= \frac{4\pi R^3}{3\sqrt{3}}$$

$$= \frac{4\pi\sqrt{3}R^3}{9}$$

(iv)

$$V_{sphere} = \frac{4}{3}\pi R^3$$

$$\frac{V_{cylinder}}{V_{sphere}} = \frac{4\sqrt{3}\pi R^3}{9} \div \frac{4}{3}\pi R^3$$

$$= \frac{4\sqrt{3}\pi R^3}{9} \times \frac{3}{4\pi R^3}$$

$$= \frac{3\sqrt{3}R^3}{9R^3}$$

$$= \frac{\sqrt{3}}{3}$$

$$= \frac{1}{\sqrt{3}}$$

Therefore the ratio of $V_{cylinder} : V_{sphere} = 1 : \sqrt{3}$